

## 確認テスト 1 (極限、微分編)

1. 以下の極限を求めなさい

$$(1) \lim_{x \rightarrow \infty} \sqrt{x}(\sqrt{x+1} - \sqrt{x})$$

$$(2) \lim_{x \rightarrow -\infty} (\sqrt{x^2 + x} + x)$$

2. 以下の関数を微分しなさい

$$(1) f(x) = \frac{2x}{x^2 - 1}$$

$$(2) f(x) = \sqrt{\frac{x^2 - 9}{x + 8}}$$

## 解答

1. 以下の極限を求めなさい

$$\begin{aligned}(1) \lim_{x \rightarrow \infty} \sqrt{x}(\sqrt{x+1} - \sqrt{x}) &= \lim_{x \rightarrow \infty} \frac{\sqrt{x}(\sqrt{x+1} - \sqrt{x})(\sqrt{x+1} + \sqrt{x})}{\sqrt{x+1} + \sqrt{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{x}}{\sqrt{x+1} + \sqrt{x}} \\ &= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{x}} + 1} \\ &= \frac{1}{2}\end{aligned}$$

(2)  $x = -y$  と置き換えると

$$\begin{aligned}\lim_{x \rightarrow -\infty} (\sqrt{x^2 + x} + x) &= \lim_{y \rightarrow \infty} (\sqrt{y^2 - y} - y) \\ &= \lim_{y \rightarrow \infty} \frac{-1}{\sqrt{1 - \frac{1}{y}} + 1} \\ &= -\frac{1}{2}\end{aligned}$$

2. 以下の関数を微分しなさい

$$\begin{aligned}(1) f'(x) &= 2 \frac{(x^2 - 1)x \cdot 2x}{(x^2 - 1)^2} \\ &= \frac{-2(x^2 + 1)}{(x^2 - 1)^2}\end{aligned}$$

$$\begin{aligned}(2) f'(x) &= \left\{ \left( \frac{x^2 - 9}{x + 8} \right)^{\frac{1}{2}} \right\}' \\ &= \frac{1}{2} \sqrt{\frac{x + 8}{x^2 - 9}} \cdot \frac{2x \cdot (x + 8) - 1 \cdot (x^2 - 9)}{(x + 8)^2} \\ &= \frac{x - 2 + 16x + 9}{2\sqrt{(x^2 - 9)(x + 8)^3}}\end{aligned}$$