

確認テスト 2 (極限、微分編)

1. 以下の極限を求めなさい

$$(1) \lim_{x \rightarrow \infty} x \left(3 - \sqrt{9 - \frac{2}{x}} \right)$$

$$(2) \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{\sqrt{1-x}} - \frac{2}{\sqrt{4-x}} \right)$$

2. 以下の関数を微分しなさい

$$(1) f(x) = (2x - 1)(x^2 - x + 3)$$

$$(2) f(x) = \frac{2x - 3}{x^2 + 1}$$

解答

1. 以下の極限を求めなさい

$$\begin{aligned}(1) \lim_{x \rightarrow \infty} x(3 - \sqrt{9 - \frac{2}{x}}) &= \lim_{x \rightarrow \infty} \frac{x\{3^2 - (9 - \frac{2}{x})\}}{3 + \sqrt{9 - \frac{2}{x}}} \\ &= \lim_{x \rightarrow \infty} \frac{2}{3 + \sqrt{9 - \frac{2}{x}}} \\ &= \frac{1}{3}\end{aligned}$$

$$\begin{aligned}(2) \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1}{\sqrt{1-x}} - \frac{2}{\sqrt{4-x}} \right) &= \lim_{x \rightarrow 0} \frac{1}{x} \cdot \frac{\sqrt{4-x} - 2\sqrt{1-x}}{\sqrt{1-x}\sqrt{4-x}} \\ &= \lim_{x \rightarrow 0} \frac{3}{\sqrt{1-x}\sqrt{4-x}(\sqrt{4-x} + 2\sqrt{1-x})} \\ &= \frac{3}{8}\end{aligned}$$

2. 以下の関数を微分しなさい

$$\begin{aligned}(1) f'(x) &= 2(x^2 - x + 3) + (2x - 1)(2x - 1) \\ &= 6x^2 - 6x + 7\end{aligned}$$

$$\begin{aligned}(2) f'(x) &= \frac{2(x^2 + 1) - (2x - 3) \cdot 2x}{(x^2 + 1)^2} \\ &= \frac{2x^2 + 2 - 4x^2 + 6x}{(x^2 + 1)^2} \\ &= \frac{-2x^2 + 6x + 2}{(x^2 + 1)^2}\end{aligned}$$